# Color Calibration of a Colorimetric Scanner Using Non-Linear Least Squares 

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#### Abstract

In this paper we present a technique to improve the calibration of a colorimetric scanner by employing an affine model in conjunction with non-linear least squares fitting.

In the affine model, the transformation from image RGB values to CIE XYZ values is specified as a multiplication with a $3 \times 3$ matrix, $M_{s}$, followed by the addition of a constant vector, $K$. This model allows one to first transform the image RGB values to CIE XYZ values under a specified illuminant and then to convert these XYZ values to the appropriate RGB or CMYK values for viewing on different output devices. The precise values of the entries in the matrix $M_{s}$ and vector $K$ are determined through a nonlinear least squares procedure, in which the total CIE $\Delta E$ between the ideal XYZ and predicted XYZ values of a calibration chart is minimized.

We interpret the role of the constant term $K$ to be a subtraction of the white of the illuminating source.


## Keywords:

scanners, color calibration, colorimetric, least squares

## Introduction

A requirement in today's world of digital scanning is that the scanned image needs to be repurposed for display on a variety of monitors and printers, and this must satisfy the dual criteria of accurate as well as aesthetically pleasing color reproduction.

One feasible solution is to first transform the image RGB values to CIE XYZ values under a specified illuminant and then to convert these XYZ values to the appropriate RGB values for viewing on different output devices. Different image (scanner) whites are preferred by viewers depending on the output device. Thus the best solution is to convert on the fly the scanned image RGB values to the XYZ values under the illuminant that is representative of the display white point. Hence, we seek a conversion from image RGB to XYZ that is fast, accurate
and imposes minimal additional overhead (no special hardware, or increased storage/memory).

## Methods

We solve this problem by modifying the conventional representation

$$
\begin{align*}
& \lceil X\rceil \quad\lceil R\rceil \\
& |Y|=\mathbf{M}_{s}|G| \\
& \lfloor Z\rfloor\lfloor B\rfloor \tag{1}
\end{align*}
$$

where $M_{s}$ is a $3 \times 3$ scanner matrix, and provides a linear mapping between RGB and XYZ values.

The simplest extension to the above model is to make it affine instead of linear through the addition of a constant term.

$$
\begin{align*}
& \lceil X\rceil\left\lceil M_{s}(1,1) \quad M_{s}(1,2) \quad M_{s}(1,3)\right\rceil\lceil R\rceil\left\lceil k_{1}\right\rceil \\
& |Y|=\left|M_{s}(2,1) \quad M_{s}(2,2) \quad M_{s}(2,3)\right|\left|{ }_{G}\right|_{+}\left|k_{2}\right| \\
& \lfloor Z\rfloor\left\lfloor M_{s}(3,1) \quad M_{s}(3,2) \quad M_{s}(3,3)\right\rfloor\lfloor B\rfloor\left\lfloor k_{3}\right\rfloor \tag{2}
\end{align*}
$$

where $\mathbf{k}=\left(k_{1} ; k_{2} ; k_{3}\right)$ may be viewed as a bias term.
The parameters comprising $\mathbf{M}_{s}$ and $\mathbf{k}$ can be determined if we have sufficient sample measurements for the corresponding $X Y Z$ and $R G B$ values. There are several techniques available to estimate these parameters. One technique is to use linear regression, and then calculate the color errors that result from the transformation [3].* An alternative is to formulate this as an optimization problem where the cost function is the total CIE $\Delta E$ over all the samples, and the optimal $\mathbf{M}_{s}$ and $\mathbf{k}$ are those that minimize this cost function.

For a given sample $i$, we can use equation 2 to express

$$
\left.\left.\begin{array}{l}
\left\lceil X_{i}\right\rceil\left\lceil M_{s}(1,1)\right. \\
M_{s}(1,2)
\end{array} M_{s}(1,3)\right\rceil\left\lceil R_{i}\right\rceil\left\lceil k_{1}\right\rceil\right] \text {. }
$$

where $X_{i}, Y_{i}$ and $Z_{i}$ are the $X Y Z$ values for the $i$ th sample
predicted by the model from the $R G B$ values for that sample.

Let $X_{w}, Y_{w}$ and $Z_{w}$ be the XYZ coordinates of the white point of the scanner (e.g. the $X Y Z$ values for D50). Then $X_{i}$, $Y_{i}$ and $Z_{i}$ can be transformed into $L^{*} a^{*} b^{*}$ values using the well known equations [6, pg.167].

Let $X_{s i}, Y_{s i}$ and $Z_{s i}$ be the ideal, spectrally measured $X Y Z$ values for the $i$ th sample as measured by a spectrometer. We can obtain the ideal $L^{*} a^{*} b^{*}$ values using the above equations. The difference is calculated using $\Delta L^{*}{ }_{i}=L^{*}{ }_{s i}-$ $L^{*}, \Delta a_{i}{ }_{i}=a^{*}{ }_{s i}-a^{*}{ }_{i}$, and $\Delta b_{i}^{*}=b^{*}{ }_{s i}-b_{i}^{*}$.

The CIE metric for the color difference is

$$
\begin{equation*}
\Delta E_{i}=\sqrt{\frac{\Delta L *_{i}^{2}}{4}+\Delta a *_{i}^{2}+\Delta b *_{i}^{2}} \tag{4}
\end{equation*}
$$

The objective function that we seek to minimize is

$$
\begin{equation*}
S=\bullet \Delta E_{i} \tag{5}
\end{equation*}
$$

over all the samples in the test chart.

## Results

This optimization problem was solved using MATLAB on an IBM-RS/6000 platform running AIX. Specifically, the function leastsq from the Optimization toolbox was used with the Levenberg-Marquardt method. The starting guess is obtained by setting $\mathbf{M}_{\mathrm{s}}$ to be the solution to the linear least squares problem (ie without the constant term) and by setting the vector $\mathbf{k}$ to zero. ${ }^{\dagger}$

We used a Kodak-Q60 IT-8 color test chart. The Gretag Spectroscan instrument [1] was used to meausure the spectral re ectance of the samples in this chart. The illuminant was D50. These can be converted to XYZ values by integrating the product of the spectral reflectance, the spectral distribution of the light (for D50), and the CIE $\bar{x}, \bar{y}, \bar{z}$ functions. The IBM TDI/Pro 3000 colorimetric

scanner described in [4] was used to measure the corresponding RGB values.

The result of the optimization is given below:

$$
\mathbf{M}_{\mathbf{s}}=\left[\left.\begin{array}{ccc}
1.0676 & -0.2307 & 0.1355  \tag{6}\\
0.0930 & 0.8877 & 0.0339 \\
-0.0557 & 0.0939 & 0.7757
\end{array} \right\rvert\,\right.
$$

and

$$
\mathbf{k}=\left[\begin{array}{l}
\lceil-16.8750\rceil  \tag{7}\\
-15.0017 \\
-11.1541 \\
\hline
\end{array}\right.
$$

We obtained an average $\Delta E=1.6212$ and maximum $\Delta E$ $=4.549$ over the same Kodak Q60 chart. In order to test the performance of our method, we measured and scanned a mini-Macbeth chart using the same Gretag instrument and scanner. The above values of $\mathrm{M}_{\mathrm{s}}$ and k were used to predict the $X Y Z$ values. We obtained an average $\Delta E=1.7910$ and maximum $\Delta E=5.43$.

We compared these results with a linear least squares solution, which resulted in

$$
\mathbf{M}_{\mathbf{s}}=\left[\begin{array}{ccc}
1.1998 & -0.4140 & 0.1873  \tag{8}\\
0.2059 & 0.7289 & 0.0775 \\
\lfloor 0.0450 & -0.0246 & 0.7999 \\
\hline
\end{array}\right.
$$

We obtained an average $\Delta E=2.332$ and maximum $\Delta E$ $=11.945$ over the same Kodak Q60 chart. On the miniMacbeth chart we obtained an average $\Delta E=2.458$ and maximum $\Delta E=6.72$.

The chromaticity plot for the Kodak Q60 chart is shown in Figure 1.

The histogram plots of $\Delta E$ for the Kodak Q60 chart is shown in Figure 2.

Figure 1. CIE xy chromaticity plots for measurements made on the Kodak Q60-IT8 test chart. (a) is for linear least squares, and (b) is for non-linear least squares.


Figure 2. Histogram for $\Delta E$ values measured on the Kodak Q60-IT8 test chart. (a) is for linear least squares, and (b) is for non-linear least squares.

## Discussion

By using the affine model with non-linear least squares fitting, the average $\Delta E$ is lowered by $27 \%$ and the maximum $\Delta \mathrm{E}$ is lowered by $19.6 \%$ for the mini-Macbeth chart, and the corresponding values for the Kodak Q60 chart are $30 \%$ and $38 \%$ respectively. These are significant reductions in the error.

The interpretation we offer for the constant term $k$ is that it represents the white of the illuminating source. Indeed, if we display the constant term $\mathbf{k}$ from equation 7 on a calibrated monitor, the color appears pinkish, that of the illuminant D50 used to generate the ideal $X Y Z$ values. Furthermore, if we change the illuminating source from D50 to D45, D55, D60, and D65, and redo the entire calibration calculations, the locus of chromaticity coordinates of the constant $\mathbf{k}$ follows the locus of the illuminants.

Thus, the color correction that is applied due to the constant term $\mathbf{k}$ can be viewed as a subtraction of white.

## Conclusion

The solution we have proposed does demonstrably better than linear least squares, with only three additional subtractions per pixel, and no additional storage/memory requirements. Though we can use other techniques such as neural networks [5, 2] or LUTs, they impose additional computational overhead at the time of conversion of RGB to XYZ. The size of the LUTs needs to be either very large, or interpolation must be done on the fly for every pixel value.

Thus we have provided a model that does better than linear least squares with the smallest overhead in
repurposing the image. We also provided a physical justification for the terms appearing in our model.

* Note that during the regression step, no explicit $\Delta \mathrm{E}$ measurements are made.
$\dagger$ We varied the initial guess and found it did not change the solution to the optimization problem.


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